Solutions:

1. (14 points) Dr. Frankenstein is growing two types of super-bacteria in his secret lab: A and B.
   - Bacteria A's population grows by 10% every hour. At midnight, he had 5000 bacteria of type A.
   - Bacteria B's population triples every 5 hours. At 1:00 AM, he had 1000 bacteria of type B.

   When will Dr. Frankenstein have twice as many bacteria B as bacteria A? Round to the nearest minute.

With t = number of hours since midnight:

$\boxed{A(t) = 5000\,(1.1)^t}$ (or $\cong 5000\, e^{0.09531018t}$)

$\boxed{B(t) = \left(\frac{1000}{\sqrt[5]{3}}\right)\left(\sqrt[5]{3}\right)^t}$ (or $\cong 802.7416\, e^{0.219722458t}$)

$\cong (802.7416)\cdot(1.24573094)^t$

We can get these by modeling $A(t) = A_0 a^t$ or $A_0 e^{at}$

using: $A(0) = 5000$, $A(1) = 5500$ ($= 5000 + 10\%\, 5000$)

and similarly for $B(t) = B_0 b^t$

using: $\underbrace{B(1)}_{B_0 b^1} = 1000$ & $\underbrace{B(6)}_{B_0 b^6} = 3(1000) = 3000$.

$B_0 b^1 = 1000$ & $B_0 b^6 = 3000$

Dividing: $\frac{\cancel{B_0} b^6}{\cancel{B_0} b} = 3 \Rightarrow b^5 = 3 \Rightarrow b = \sqrt[5]{3}$.

Replacing: $B_0 b^1 = B_0 \sqrt[5]{3} = 1000 \Rightarrow B_0 = 1000/\sqrt[5]{3} \cong 802.7416$

Want: $B(t) = 2A(t)$ i.e. $802.7416\,(\sqrt[5]{3})^t = 2\left[(5000)(1.1)^t\right]$

Solve: $802.7416\,(\sqrt[5]{3})^t = 10{,}000\,(1.1)^t$

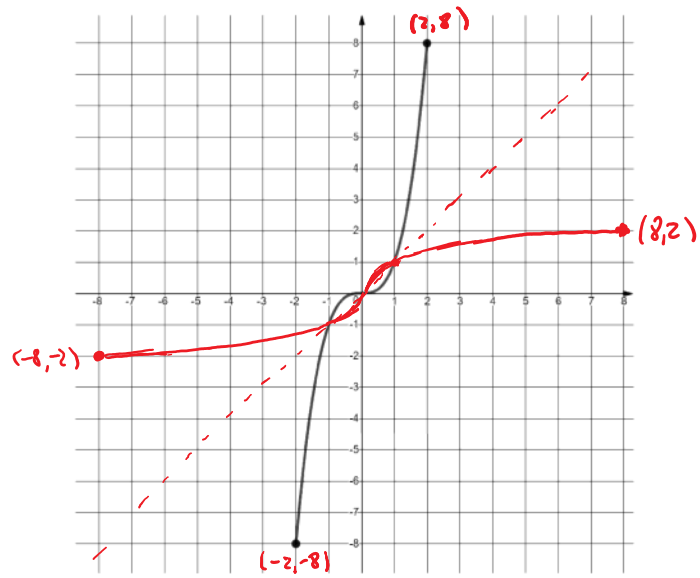
$0.08027416 = \left(\frac{1.1}{\sqrt[5]{3}}\right)^t$

$\ln(0.08027416) = t \ln\left(\frac{1.1}{\sqrt[5]{3}}\right)$

$t = \frac{\ln(0.08027416)}{\ln(1.1/\sqrt[5]{3})} = \boxed{20.27378\ldots} \cong \boxed{8:16\text{ PM}}$

hrs past midnight

2. (6 points) The following is the graph of a function $F(x)$, with domain $-2 \leq x \leq 2$.
On the same grid, sketch the graph of its inverse function, $F^{-1}(x)$. Also, specify its domain and range.



The domain of F^{-1} is: $[-8, 8]$, and the range of F^{-1} is: $[-2, 2]$

3. (6 points) Consider the function $G(x) = -3x^2 + 18x$, with domain $x \geq 3$. Find the rule for $G^{-1}(x)$.

$y = -3x^2 + 18x \quad , \quad x \geq 3$

$x = -3y^2 + 18y$

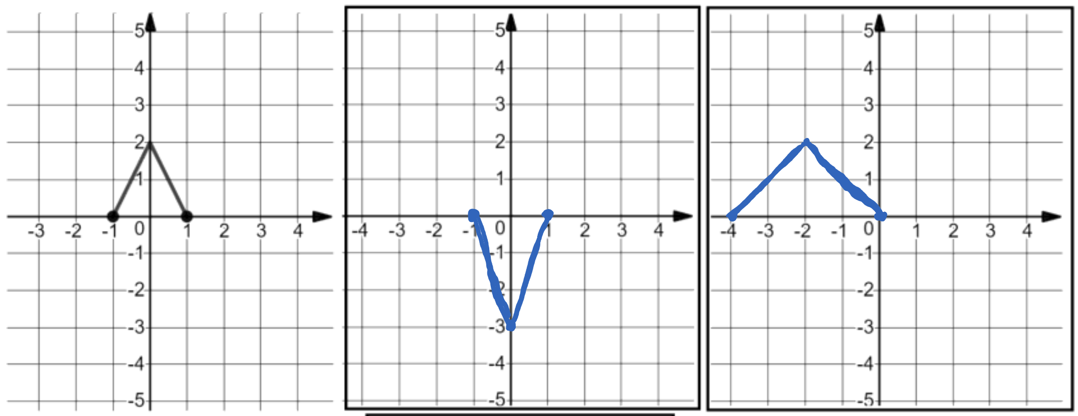
$3y^2 - 18y + x = 0$

Q.F. $y = \dfrac{18 \pm \sqrt{324 - 12x}}{6} = 3 \pm \dfrac{\sqrt{324-12x}}{6} \geq 3$

$$\boxed{G^{-1}(x) = \frac{18 + \sqrt{324-12x}}{6} = 3 + \frac{\sqrt{324-12x}}{6}}$$

4. The graph of a function $y = f(x)$ with domain $-1 \le x \le 1$ is shown on the left below.

(a) (6 points) Draw the graphs of the two indicated functions in the boxed areas below.
Use the first grid for scratch work or intermediate steps – be neat and clear in the boxed areas.



Graph of $y = f(x)$

1. **Sketch the graph of $y = -\frac{3}{2}f(x)$**

Vertical Reflection
Vertical expansion $y \cdot \frac{3}{2}$

2. **Sketch the graph of $y = f(0.5x + 1)$**

1) Horizontal shift LEFT by 1
2) Horizontal dilation $x \cdot 2$

(b) (6 points) For the same function $f(x)$ shown above left, determine the constants A, B for which the range of $Af(x) + B$ is $[-1, 3]$.

From the graph of $f(x)$: $0 \le f(x) \le 2$

Assuming $A > 0$: $\underbrace{A(0) + B}_{=-1} \le A f(x) + B \le \underbrace{A(2) + B}_{=3}$

$$\begin{cases} B = -1 \\ 2A + B = 3 \Rightarrow 2A - 1 = 3 \Rightarrow A = 2 \end{cases}$$

So: $\boxed{A = 2,\ B = -1}$ (vertical dilation times 2 followed by shift down by 1)

5. (12 points) Solve the following equations. Show your steps and box your final answer.

(a) $3 \cdot 5^{2x} - 7 = 1$

$3 \cdot 5^{2x} = 8$

$5^{2x} = 8/3$

$(2x)\ln(5) = \ln(8/3)$

$\boxed{x = \frac{\ln(8/3)}{2\ln 5} \approxeq 0.3047}$

(b) $\ln(x^2 - 3) = 0$

$x^2 - 3 = e^0 = 1$

$x^2 = 4$

$\boxed{x = \pm 2}$

(c) $\log_2(x+1) - \log_2(x) = 3$

$\log_2\left(\frac{x+1}{x}\right) = 3$

$\frac{x+1}{x} = 2^3 = 8$

$x + 1 = 8x$

$7x = 1$

$\boxed{x = 1/7} \approxeq 0.14$